THE UNIVERSITY OF HONG KONG

Department of Mathematics

COLLOQUIUM

Intersection of Schubert varieties and inequalities for eigenvalues of sums of self-adjoint operators

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Abstract

Consider self-adjoint operators A, B, C: $\mathcal{H} \to \mathcal{H}$ on a finite-dimensional Hilbert space such that $A + B + C = 0$. Let $\{\lambda_j(A)\}$, $\{\lambda_j(B)\}$, and $\{\lambda_j(C)\}$, be sequences of eigenvalues of A, B, and C counting multiplicity, arranged in decreasing order. In 1962, A. Horn conjectured that the relations of $\{\lambda_j(A)\}$, $\{\lambda_j(B)\}$, and $\{\lambda_j(C)\}$ can be characterized by a set of inequalities defined inductively. This problem was eventually solved by A. Klyachko and Knutson-Tao in the late 1990s. (Two excellent articles on the subject are: W. Fulton, Eigenvalues, invariant factors, hightest weights, and Schuber calculus, *AMS Bulletin* 37 (2000), pp. 209-249, and A. Knutson and T. Tao, Honeycombs and Sums of Hermitian Matrices, *AMS Notices* 48 (2001), pp. 175 - 186.) In this talk we will show that these inequalities are also valid for selfadjoint elements in a finite factor. The major difficulty in our argument is the proof that certain generalized Schubert cells have nonempty intersection. In the finite dimensional case, it follows from the classical intersection theory. However, there is no available intersection theory for von Neumann algebras. Our argument requires a good understanding of the combinatorial structure of honeycombs, and produces an actual element in the intersection algorithmically, and it seems to be new even in finite dimensions.

Date:	**March 13, 2009 (Friday)**
Time:	**4:00 – 5:00pm**
Place:	**Room 517, Meng Wah Complex, HKU**

All are welcome